Restricted Limits on Natural Functions with Arithmetical Graphs

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Abstract

In this paper we consider the process of defining natural functions by the operation of infinite limit: $F(\bar{x}) = \lim_{y\to\infty, y\in A} f(\bar{x}, y)$ (also limes inferior and limes superior are taken into account). But two restrictions are assumed: the given natural function f has a graph belonging to some stage of an arithmetical hierarchy, the index of a limit runs only through a given arithmetical subset A of natural numbers.

We investigate the arithmetical class of the graph of the function F, where the respective classes of the graph of f and the set A are known. The corollary for the Turing degrees of F is formulated.

Keywords: Theory of computation, Infinite limits.

1 Introduction

The operation of infinte limits (including limes inferior and limes superior) is a natural operation on functions. The main field for the limit operation is in the mathematical analysis (for the real functions). But also the case of natural functions is considered in mathematics and computer science (for example the important Shoenfield's Limit Lemma [11]).

In this paper we use the limit operation as an 'ideal' component of computing systems. Some existing models of computation have a strong connection with the mathematical analysis and its tools. The best example is Shannon's General Purpose Analog Computer [10].

The General Purpose Analog Computer (GPAC) is a computer whose computation evolves in continuous time. The outputs are generated from the inputs by means of a dependence defined by a finite directed graph (not necessarily acyclic) where each node is one of the following boxes: *integrator*: a two-input, one-output unit with a setting for initial condition, if the inputs are unary functions u, v, then the output is the Riemann-Stieljes integral $\lambda t.\int_{t_0}^{t} u(x)dv(x) + a$, where a and t_0 are real constants defined by the initial settings of the integrator; *constant multiplier*: a one-input, one-output unit associated to a real number, if u is the input of a constant multiplier associated to the real number k, then the output is ku; *adder*: a two-input, one-output unit, if u and v are the inputs, then the output is $u + v$; *multiplier*: a two-input, one-output unit, if u and v are the inputs, then the output is uv; *constant function*: a zero-input, one-output unit, the value of the output is always 1.

Rubel in his papers [7, 8] extended this model by an introduction of new boxes to define Extended Analog Computer in the real realm. This model is similar to the GPAC but it allows, in addition, other types of units, e.g. units that solve boundary value problems

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